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Private Equity Recommitment Strategies for Institutional Investors

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Institutional investors must deal with irrevocable commitments, cash flow uncertainty, and illiquidity when making new commitments to maintain their portfolio exposure to private equity funds. This study develops a dynamic recommitment strategy to preserve the strategic allocation to private equity. For each period, the level of new commitments is determined by characteristics of the existing private equity portfolio, including received distributions, uncalled capital from old commitments, and the current allocation relative to its target level.

Today, private equity is included in the investment portfolios of many endowments, foundations, pension funds, and insurance companies. These institutional investors typically set a specific allocation for private equity as part of their strategic policy portfolio (Sharpe 2010). A large majority of institutional investors fulfill this allocation indirectly through private equity "funds" rather than through direct private equity investments, presumably because successfully entering, managing, and exiting direct private equity investments is generally recognized as requiring a high level of expertise and experience, which many institutional investors lack.[1] Investors provide capital for private equity funds and rely on the funds' management to decide when and which investments are made.[2] These stakes in private equity funds are illiquid owing to restrictions on the sale of such investments (see Sahlman 1990; Lerner and Schoar 2004) and the lack of a well-developed secondary market.[3] Thus, investors gain exposure to private equity primarily by participating in new funds, whereby they commit themselves for a certain amount of capital. These commitments are invested ("called") gradually over a period of several years and at the discretion of the funds' management. Very often, not all committed capital is called. Moreover, payouts from disinvestments ("distributions") typically start when a fund is only a few years old, often before all committed capital has been invested. Unfortunately, in most cases these distributions cannot be reinvested immediately but must be recommitted to new private equity funds. In short, the cash inflows and outflows of private equity funds are uncertain and are not controlled by the investor, which can lead to private equity misallocation in the portfolios of institutional investors.

The unpredictable cash flows, together with the illiquidity of the market, create a challenge for institutional investors to keep their private equity investments at the desired level. This issue has received very little attention in the literature despite the fact that the costs of inefficient (re)commitment can be huge. On the one hand, being underinvested in private equity (because part of the committed capital is not yet invested) can lead to a performance drag on the portfolio, given the private equity return profile. On the other hand, investors can become overinvested by making commitments that are too large, which can result in a liquidity shortfall whereby investors do not have the cash to honor a new capital call. Penalties for investors who fail to meet their capital call obligations are often draconian. In such cases, the investor typically faces involuntary liquidation by instantly losing the economic value of her existing fund investment, which is distributed among the other participants in the fund. Moreover, cases of additional claims against the defaulting investor for losses resulting from the default have been reported. Clearly, investors do their utmost to avoid this situation and would rather sell their fund investment in the secondary market. A forced sale usually involves such a large discount that this exit strategy is an option of last resort. In 2008, the

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liquidity crisis and its aftermath taught us that liquidity shortfall can be a serious concern for institutions with high allocations to illiquid asset classes (Siegel 2008). An efficient recommitment strategy could thus mitigate opportunity costs, in the case of underinvesting, and cash shortages, in the case of overinvesting, which eventually result in economic losses. A closer examination of the issues surrounding the maintenance of the target level of exposure to private equity via indirect investment in private equity funds could thus be of great help to institutional investors.

In our study, we designed a recommitment strategy that enables institutional investors to maintain a private equity fund portfolio that matches their strategic target allocation. Institutional investors face a multiperiod dynamic portfolio optimization problem in which each period requires a decision on new commitments that affects the level of investments in all future periods. We used the solution to the corresponding single-period problem to develop our recommitment strategy. The key feature of our strategy is that the level of new commitments in a given period depends on the characteristics of the current private equity portfolio. Using our strategy, one can make new commitments that are equal to last quarter's received distributions plus the uncalled capital from old commitments, scaled by the ratio of the target allocation to the current private equity allocation. Recommitting the cash distributions is intuitive because these liquidated investments should be reinvested in private equity as soon as possible to keep the allocation at the desired level. Reallocating uninvested commitments after a certain period of time is necessary to prevent diminution of private equity exposure, especially since, in many cases, not all committed capital is called. Finally, the scaling factor, which comprises characteristics of the existing portfolio, is used to either boost or temper the amount of new commitments to bring the exposure to the desired level. The dynamic nature of our strategy is in sharp contrast to that of Cardie, Cattanach, and Kelley (2000) and Nevins, Conner, and McIntire (2004), who considered static strategies that allocate private equity investments according to a predetermined set of fixed rules rather than incorporate characteristics of the existing portfolio.[4]

■ *Discussion of findings.* Using the Thomson Venture Economics database, we empirically evaluated our recommitment strategy by means of historical simulations over 1980–2005. Although, in practice, private equity investments are often part of a larger portfolio that also includes other assets, such as stocks and bonds, we focused on private equity allocation; thus, we looked at only 100% private equity portfolios to avoid the undue influence of the dynamics of other asset classes on our simulation results. Our main finding is that our dynamic recommitment strategy can maintain a stable private equity fund portfolio that reaches an average level of 86% of the target allocation while keeping the probability of being overinvested to a low 8%. Our sensitivity analyses show that this strategy remains equally successful when the portfolio is restricted to a particular private equity segment (buyout or venture capital), a specific region (the United States or Europe), or varying fund manager experience (first-time or follow-on funds). Furthermore, we found that achieving the target exposure is possible only when commitments during the buildup phase of a new portfolio (in our case, one year) are 30% higher than the desired strategic allocation ("overcommitment"). The reason is that disinvestments occur before the final investments are made and 10% of the committed capital, on average, is never invested by the private equity fund. Moreover, investors with enough liquidity for a (temporarily) higher allocation should also consider overcommitment for their recommitments. Setting this additional overcommitment to 30% brings the average portfolio exposure closer to the target (98%), albeit at the cost of a substantially higher probability of being overinvested (43%). For this strategy, the 99% confidence interval for the realized allocation to private equity is between 70% and 141% of the target allocation.

Data

We obtained our dataset from Thomson Venture Economics (TVE). We used quarterly contributions, distributions, and net asset values (NAVs) for 2,786 individual private equity funds over 1980:Q1–2005:Q4. Reported cash flows are in U.S. dollars and are net of management fees, performance fees ("carried interest"), and other costs (e.g., taxes). We thoroughly inspected the dataset for errors and mistakes, which led us to make a few data corrections and to exclude some funds (for details, see Appendix A). We ended up with 2,617 funds for our empirical analysis. We included several fund characteristics: regional focus (the United States or the EU), investment type (venture capital [VC] or buyout capital [BO]), fund manager experience (first-time or follow-on), and year of the fund's formation ("vintage year"). **Table 1** reports the distribution of funds with respect to investment type, fund manager experience, and region. Approximately two-thirds of all funds were venture capital funds, and about 60% were U.S.-based funds. Our sample included 42% first-time funds and 58% follow-on funds.

Table 1. Distribution of Private Equity Funds across Investment Types, Manager Experience, and Regions

	United States		Europe		Total	
Venture capital	1,089		591		1,680	
First-time		384		303		687
Follow-on		705		288		994
Buyout capital	535		402		937	
First-time		244		158		402
Follow-on		291		244		535
Total	1,624		993		2,617	

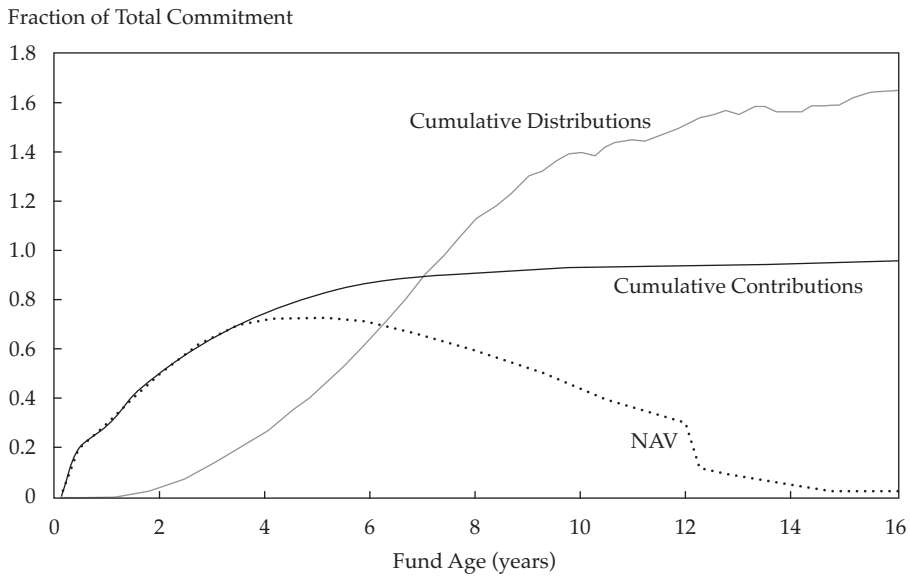
Our dataset is comparable to that of Jones and Rhodes-Kropf (2003), Kaplan and Schoar (2005), and Phalippou and Gottschalg (2009), to which we refer readers interested in more information regarding the way TVE collects its data. These studies also discuss potential biases in the TVE database that arise mostly from the fact that it is based on voluntary reporting by private equity funds and may exhibit stale pricing. Because of our study's setup (i.e., we focused on 100% private equity portfolios), these selection and valuation biases likely did not affect our conclusions.[5]

Note that our dataset includes no information on the covenants in the agreements between the private equity funds and their institutional investors. That said, two covenants related to the overall fund management could affect our strategies because they influence the timing of a fund's cash flows: restrictions on both co-investments[6] and reinvestments. Co-investing entails investments in the same investee company by multiple funds that are managed by the same firm. As discussed by Gompers and Lerner (1996, 1999), this scenario is undesirable because it could lead to such opportunistic behavior as bailing out poor investments of another fund of the firm. Agreements for second or later follow-on funds often impose restrictions on co-investing. Nevertheless, Cumming, Schmidt, and Walz (2010) observed co-investments in 21% of their analyzed venture capital deals. In our robustness analysis (discussed later in the article), we split our sample into first-time and follow-on funds, which allowed us to test the impact of co-investing because first-time funds are not subject to co-investing from earlier funds of the same management firm. Reinvestment concerns reinvestment of the capital gains of a fund disinvestment back into the same fund, which may yield further profits for the fund's investors. Reinvestments affect the NAV (because more capital is available for investment) and the timing of the cash flows: No capital is called from investors, and distributions to investors will occur later. Because of the structure of our dataset, which includes only realized distributions, we were unable to measure the magnitude and importance of reinvestments. Apart from the fact that reinvestments could lead to an extension of a fund's life, they would be unlikely to affect our results materially. Moreover, fund covenants usually allow managers to reinvest proceeds from disinvestments within a year after the investment is completed.

Summary Statistics. Investments in private equity funds start with the institutional investor's committing a certain amount of capital. No capital is actually exchanged when this decision is made, but from that moment on, the investor must provide capital whenever the fund manager asks for it. The commitments are irrevocable during the lifetime of the fund, which typically ranges between 10 and 14 years. In this period, the fund manager independently decides on the fund's investments and disinvestments. As investment opportunities arise, part of the committed capital will be called by the fund manager. These contributions include the capital that is actually invested as well as fees.[7] Private equity funds generally unwind their funds by distributing the proceeds of disinvestments to the investors.

Figure 1 shows the average cumulative cash flows (contributions and distributions) over the lifetime of the funds in our dataset. We scaled the cash flows by the total commitments to each fund to make them comparable across funds and independent of fund size.[8] Note that it takes several years for the committed capital to be invested. Although investments are largest in the first year of the fund's lifetime, only 32% of the commitments, on average, are called. Additional capital is invested in subsequent years but at a gradually declining pace. In the second year, on average, 19% of the commitments are called, followed by 15%, 10%, 7%, and 5% in the next four years. After approximately six years, cumulative contributions level off. Note that, on average, only about 90% of total commitments are eventually called by the private equity fund. Put differently, 10% of the original commitments are never invested.

Figure 1. Average Cumulative Contributions, Average Cumulative Distributions, and Average NAVs of Individual Private Equity Funds, 1980:Q1–2005:Q4



The average cumulative distributions exhibit a typical S-shape. Starting after two years, distributions are made at an accelerating pace until the funds reach the age of seven to eight years. At that point, the cumulative distributions are approximately equal to the total commitments. Subsequently, payouts start to decline, until cumulative distributions eventually level off at around one and a half times the total commitments after 12 years.

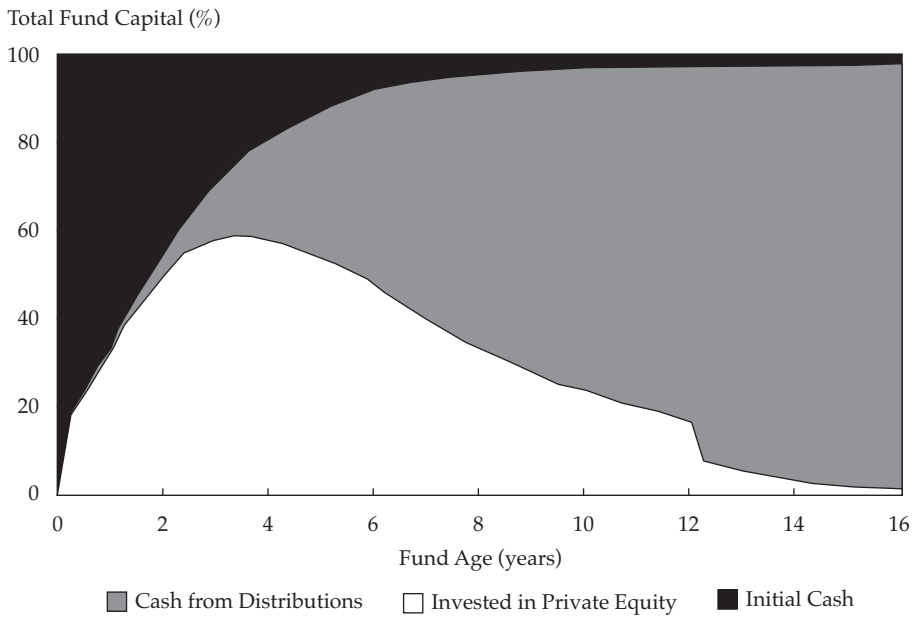
Figure 1 also shows the average value of investments over the fund's lifetime, again expressed as a fraction of total commitments. Because of the patterns of contributions and distributions, the NAV builds up quickly during the first few years of the fund's lifetime, reaches its maximum between four and six years, and then gradually drops off again. The average NAV apparently does not decline to zero completely even after 15 years because some funds keep a residual value although they show no signs of activity (as mentioned earlier, the lifetime of a typical private equity fund ranges between 10 and 14 years). Following Ljungqvist and Richardson (2003), who suggested that these residual values are unreliable, we set the NAV equal to zero after 12 years if there were no signs of activity at that point or after the last recorded activity if any cash flows occurred in Year 13 or later. The effect of this write-off rule is observable in the NAV at the end of Year 12 in Figure 1.[9]

The key concept in our recommitment strategies is the investment degree, which measures the actual exposure of a private equity fund investment. Specifically, we defined the investment degree as the fraction of total capital that is actually invested in private equity—that is, the investment degree at the end of quarter t (ID_t) is defined as

$$ID_t = \frac{NAV_t}{NAV_t + Cash_t}, \tag{1}$$

where NAV_t is the sum of the NAVs of the underlying private equity fund investments and $Cash_t$ is the uninvested cash in the portfolio. Obviously, an investor aims for an investment degree equal to 1 or, equivalently, for the amount of cash to be equal to zero. In practice, achieving this target of being fully invested is complicated. Commitments are only gradually called to be invested, and distributions occur before all committed capital has been invested; thus, the amount of cash will typically be positive. The impact of these dynamics on the investment degree and, thus, on private equity exposure is depicted in **Figure 2**, which shows the cash from the initial commitment, the cash from distributions, and the NAV of the actual private equity investments as a fraction of total capital over a fund's lifetime. From Figure 2, we can see that actual invested capital falls short of total capital: The investment degree reaches its maximum in the fourth year of the fund's lifetime, when it equals not more than 60%. Hence, at that point, 40% remains in cash. At all other times, private equity exposure is even less. Clearly, this outcome is undesirable for institutional investors.

Figure 2. Cash from the Initial Commitment, Cash from Distributions, and the Value of the Actual Private Equity Investments (NAV) as a Fraction of Total Fund Capital, 1980:Q1–2005:Q4



Vintage Year Statistics. Because private equity funds have a limited lifetime, most managers introduce a new fund every three to four years. All funds that start in a specific calendar year are said to belong to the same "vintage." The summary statistics discussed previously mask a great deal of variation in the cash flows and NAVs across vintage years. This is borne out by **Table 2**, which presents the maximum investment degree and its timing (in quarters after the initiation of the fund), as well as the number of funds for each vintage year in our sample period (1980–2005). The results for vintage years 2001–2005 must be treated with caution because these funds are considered immature.

The increasing number of funds per vintage year in Table 2 illustrates the remarkable growth in private equity: from 22 funds that were started in 1980 to 301 in 2000. The peaks in the number of new funds occurring at the end of the 1980s and 1990s indicate the cyclical pattern in supply and demand for private equity capital. This pattern is further illustrated by the steep decline in the number of new funds after the collapse of the dot-com bubble in 2001. The variation in the level of cumulative contributions and distributions after 16 quarters across vintages, shown in the last two columns of Table 2, also exhibits a cyclical pattern. Furthermore, the time it takes to reach the maximum investment degree varies substantially, between 11 and 23 quarters. We can infer that it takes more time to reach the maximum investment degree for funds that were started during economic downturns, as in 1990–1991 and 2000–2001.

The magnitude of the maximum investment degree exhibits a downward trend. Although it amounts to about 80% for funds initiated in the early 1980s, funds begun in the second half of the 1990s achieved a maximum investment degree of only about 60%. The sharp contrast between the maximum investment degrees in the 1980s and 1990s is mostly due to variations in the pace of the distributions. After four years, distributions range from a low of 7% for funds begun in 1982 to a high of 107% for funds dating from 1996 (the last two columns of Table 2). Clearly, the 1996 investments benefited from the investment technology hype in 1999–2000. Nevertheless, averaging per decade, we can see that the total distributions after four years are two and a half times lower in the 1980s than in the 1990s (16% compared with 41%). The differences in contributions across vintage years are much less pronounced. During the 1980s, on average, 84% of the commitments were called within four years, compared with 77% for the 1990s. Hence, we conclude that the lower maximum investment degrees during the 1990s arose not because less of the committed capital was actually invested but, rather, because distributions occurred earlier.[10]

The variation in the size and timing of the cash flows of private equity funds motivated us to design a dynamic recommitment strategy that

Table 2. Timing and Magnitude of Maximum Investment Degree across Vintage Years

Vintage Year	No. of Funds in Vintage Year	Maximum Investment Degree		$CC_{t=4y}$	$CD_{t=4y}$
		Mean	Timing		
1980	22	0.72	Q10	0.83	0.32
1981	24	0.85	Q14	0.90	0.09
1982	29	0.85	Q13	0.88	0.07
1983	63	0.83	Q13	0.92	0.23
1984	82	0.77	Q16	0.87	0.13
1985	76	0.75	Q9	0.92	0.26
1986	70	0.71	Q15	0.83	0.16
1987	116	0.68	Q18	0.78	0.14
1988	95	0.67	Q18	0.74	0.11
1989	114	0.66	Q17	0.74	0.11
1990	67	0.67	Q18	0.78	0.18
1991	61	0.55	Q17	0.63	0.15
1992	58	0.69	Q13	0.82	0.35
1993	94	0.57	Q12	0.75	0.43
1994	105	0.62	Q14	0.77	0.29
1995	111	0.61	Q17	0.77	0.38
1996	104	0.62	Q14	0.81	1.07
1997	180	0.64	Q12	0.81	0.73
1998	213	0.65	Q9	0.81	0.40
1999	248	0.57	Q12	0.75	0.12
2000	301	0.55	Q23	0.65	0.11
2001	172	0.54	Q20	0.59	0.13
2002	86	0.39	Q16	0.49	0.11
2003	60	0.61	Q12	—	—
2004	46	0.36	Q8	—	—
2005	21	0.15	Q4	—	—
Average: 1980s	69	0.75	Q14	0.84	0.16
Average: 1990s	124	0.62	Q14	0.77	0.41

Notes: For each vintage year (1980–2005), this table reports the number of funds, the magnitude and timing (in quarters) of the maximum investment degree, and the cumulative contributions and distributions after four years ($CC_{t=4y}$ and $CD_{t=4y}$). The average maximum investment degrees and timing for vintage years 2001–2005 are unreliable because the maximum and its timing cannot yet be determined with certainty. Vintage year statistics are based on the average distributions, contributions, and NAVs for all funds that were started during that year.

takes into account the composition of the current portfolio when making new commitments to achieve and maintain the desired exposure to private equity.

Dynamic Recommitment Strategies

Let us consider a hypothetical investor who wishes to maintain a constant target allocation to private equity. Although this target allocation may be part of a larger investment portfolio, we will simplify the problem by focusing on the private equity allocation only, such that our investor is assumed to own a 100% private equity portfolio. New commitments can be made every quarter, and we assume that our investor is willing to participate in each available fund.[11]

The main objective is to keep the portfolio fully invested over time—that is, to keep the investment degree, defined in Equation 1, as close as possible to 1 for all quarters t. As noted before, this is equivalent to saying that the investor aims to keep the amount of cash as close as possible to zero. Given this objective, our investor faces a multiperiod dynamic portfolio optimization problem. Each quarter requires a decision on new commitments, which affects the level of investments in all future periods given that the committed capital will be called only gradually. For simplicity, we consider the corresponding single-period decision problem for a myopic investor whose only concern is the investment degree in

the next quarter. We can then use the solution to this problem to develop dynamic recommitment strategies for long-term investments.

Single-Period Optimization Problem. Let us consider the situation where the investor must decide on the new commitments to be made at the end of quarter t, denoted C_t. The investor wishes to achieve an investment degree as close as possible to 1 in quarter $t + 1$. Although the investor does not like to be underinvested and have a large cash exposure, overinvesting should also be avoided as much as possible. Overinvesting occurs when required investments exceed the amount of available capital such that cash becomes negative (liquidity shortfall), resulting in an investment degree greater than 1. Recall that all capital calls must be met because commitments made at the start of the fund are irrevocable. Investors who are unable to meet capital call requirements face involuntary liquidation of their prior investments in the fund.

We assume that utility for our investor is symmetric such that underinvestment and liquidity shortfall entail the same costs.[12] With a symmetric loss function, our investor faces an optimization problem that can be written as

$$\min_{C_t} \mathrm{E}_t\left[\left(1 - ID_{t+1}\right)^2\right], \tag{2}$$

where ID_{t+1} is the investment degree of the private equity portfolio at the end of the next quarter[13] and E_t denotes the conditional expectation at the end of quarter t. Assuming that all capital calls from past commitments (CC_{t+1}) and distributions (D_{t+1}) occur at the end of a quarter, the next quarter's NAV (NAV_{t+1}) is given by

$$NAV_{t+1} = NAV_t\left(1 + r_{t+1}\right) - D_{t+1} + CC_{t+1}, \tag{3}$$

where r_{t+1} is the return on the private equity investments. The amount of cash at the end of quarter $t + 1$ is equal to

$$Cash_{t+1} = Cash_t + D_{t+1} - CC_{t+1}. \tag{4}$$

The total capital call at the end of quarter $t + 1$, CC_{t+1}, is a combination of the capital calls on all commitments that have been made in earlier quarters. If a fraction $\gamma_{t+1,i+1}$ of the capital committed i quarters ago, C_{t-i}, is called at $t + 1$, we have

$$CC_{t+1} = \sum_{i=0}^{T} \gamma_{t+1,i+1} C_{t-i}, \tag{5}$$

where T is the maximum fund age (in quarters) at which capital calls can be made. Note that the fractions $\gamma_{t+1,i+1}$ are not fixed *a priori* (e.g., at the start of the fund in quarter $t - i$) but are determined by the fund manager during quarter $t + 1$. Substituting Equations 3, 4, and 5 into Equation 2, we obtain

$$\min_{C_t} \mathrm{E}_t\left\{\left[1 - \frac{NAV_t\left(1 + r_{t+1}\right) - D_{t+1} + \sum_{i=0}^{T} \gamma_{t+1,i+1} C_{t-i}}{Cash_t + NAV_t\left(1 + r_{t+1}\right)}\right]^2\right\}. \tag{6}$$

Assuming that the objective function in Equation 6 is concave and that there are no further restrictions, we can take the partial derivative with respect to C_t to obtain the first-order condition

$$-2\mathrm{E}_t\left\{\left[1 - \frac{NAV_t\left(1 + r_{t+1}\right) - D_{t+1} + \gamma_{t+1,1} C_t + \sum_{i=1}^{T} \gamma_{t+1,i+1} C_{t-i}}{Cash_t + NAV_t\left(1 + r_{t+1}\right)}\right]\left[\frac{\gamma_{t+1,1}}{Cash_t + NAV_t\left(1 + r_{t+1}\right)}\right]\right\} = 0. \tag{7}$$

Rewriting Equation 7 results in the optimal commitments at the end of quarter t, given by

$$C_t = \mathrm{E}_t\left(\frac{Cash_t + D_{t+1} - \sum_{i=1}^{T} \gamma_{t+1,i+1} C_{t-i}}{\gamma_{t+1,1}}\right). \tag{8}$$

Put differently, the optimal level of new commitments is equal to the current amount of cash adjusted for the expected distributions and contributions arising from prior commitments and scaled by $\gamma_{t+1,1}$ to account for the fact that only this fraction of the new commitments will be called immediately.

Dynamic Recommitment Strategies. The solution to the single-period recommitment problem, as given in Equation 8, involves the distributions as well as the capital calls during quarter $t + 1$. Hence, implementing this strategy requires cash flow prediction models from which forecasts of these quantities can be obtained (for examples, see Takahashi and Alexander 2002; de Malherbe 2004). Pursuing a different approach, we will consider three recommitment strategies whereby the level of new commitments at the end of quarter t is determined by quantities that are available at that time. The strategies are directly motivated by the single-period optimal solution.

Strategy I simply states that distributions received during quarter t are recommitted to new private equity funds at the same time (i.e., $C_t = D_t$). In terms of the single-period solution in Equation 8,

this might be interpreted as using D_t as a sort of random-walk forecast of next quarter's distributions and neglecting the term $Cash_t - \sum_{i=1}^{T} \gamma_{t+1,i+1} C_{t-i}$, which represents the cash remaining at the end of quarter $t + 1$ owing to uncalled commitments and the scaling factor $\gamma_{t+1,1}$.

The effective investment degree in Strategy I cannot be expected to reach the target value of 1, given that committed capital will be only gradually invested over a number of years after the commitment is made. The single-period optimal solution in Equation 8 takes this factor into account through the additional term $Cash_t - \sum_{i=1}^{T} \gamma_{t+1,i+1} C_{t-i}$ (and the scaling factor $\gamma_{t+1,1}$). Although we might be tempted to replace the unknown fractions $\gamma_{t+1,i+1}$ ($i = 0, 1, \ldots, T$) with the corresponding observed fractions $\gamma_{t,i+1}$, note that doing so implies reallocating all uncalled capital from commitments made in previous quarters. In the multiperiod reality, however, this action would not be optimal because some of the earlier commitments are likely to be called after quarter $t + 1$. Hence, we adopt an alternative, but related, idea based on the observation that capital calls are typically made during the first years of a fund's lifetime.

Strategy II extends Strategy I by setting commitments at the end of quarter t equal to the sum of the current distributions D_t and the uncalled capital from commitments made P quarters ago, denoted UC_{t-P} (i.e., $C_t = D_t + UC_{t-P}$). The inclusion of UC_{t-P} is based on the observation that private equity funds usually make capital calls only during the first years of their lifetime. Capital that is not called during the first P quarters after it has been committed is no longer expected to be called and, therefore, is recommitted. In our empirical analysis, we set P equal to 24 because investment periods of private equity funds generally end after six years and any remaining commitments are unlikely to be called thereafter. This observation is confirmed by our finding that new investments level off after approximately six years, as shown in Figure 1 (see also Ljungqvist and Richardson 2003).

Although recommitting the uncalled previous commitments, as proposed in Strategy II, should help improve the average investment degree, it cannot possibly achieve the target exposure completely. As our data analysis revealed, the investment degree for individual funds reaches only 60% of committed capital, on average (see Figure 2). To counter the effects of this underinvestment and maintain the target exposure, a degree of overcommitment seems necessary. Thus, an important, but difficult, choice is the overcommitment percentage to be applied. A constant overcommitment percentage does not seem appropriate because the average (maximum) investment degree varies substantially across vintage years, as shown in Table 2. Ideally, the overcommitment percentage for new commitments in a given quarter would be based on the actual investment degree that will be attained by funds from the current vintage year, but in practice, of course, this number is unknown. Instead, we introduce a dynamic overcommitment strategy that is based on the characteristics of the current private equity portfolio and, in particular, its investment degree.

Strategy III sets the new commitments at the end of quarter t equal to the last quarter's distributions plus uncalled commitments, as in Strategy II, but then multiplied by the reciprocal of the investment degree of the current private equity portfolio—that is,

$$C_t = \frac{1}{ID_t}\left(D_t + UC_{t-P}\right). \tag{9}$$

We posit that the investment degree of the current portfolio provides valuable information regarding the appropriate overcommitment percentage for new commitments. Intuitively, the more the investment degree falls below 1, the more aggressively we should recommit capital to new private equity funds in order to bring the exposure back to the target level. Note that Strategy III will lead to "overcommitment" in the case of underinvesting and to "undercommitment" in the case of overinvesting—that is, when the current investment degree ID_t exceeds 1.

Inception of Private Equity Portfolios. These dynamic recommitment strategies implicitly assume an existing private equity portfolio. In practice, although the composition of this portfolio and its accompanying characteristics may indeed be given, that is not necessarily the case. As discussed earlier, a mature private equity portfolio cannot, in general, be bought instantaneously owing to the lack of a well-developed secondary market. Hence, the start-up of a private equity portfolio is an interesting problem in its own right. In our empirical analysis, we constructed the initial portfolio over a one-year period by making equal commitments to 16 randomly selected private equity funds with the same vintage year (four new commitments per quarter).[14] This approach is in line with that of Weidig, Kemmerer, and Born (2005), who reported that a diversified private equity portfolio contains approximately 20 funds.

As discussed previously, the average maximum investment degree of private equity funds (60%, achieved in Year 4) is well below 1, which suggests that achieving a certain level of private equity investments requires an overcommitment strategy whereby commitments exceed the available cash. For example, a commitment of 167% is required to obtain a maximum investment degree of 100% if the maximum investment degree for a 100% commitment is 60%. From our earlier discussion, we know that cash flow characteristics of private equity funds evolve over time. On the basis of these findings, we applied a 30% overcommitment in setting up the initial portfolio.

Empirical Results

Using the TVE database, we assessed the performance of the three recommitment strategies by means of historical simulation. Specifically, we formed initial portfolios of private equity funds for each vintage year by using the previously described procedure and applying the recommitment strategies for the remainder of the sample period. Several implementation issues are worth mentioning. Initially, we imposed no restrictions on the portfolio with respect to regional focus (United States or EU), investment type (VC or BO), fund management experience (first-time or follow-on funds), maximum number of funds invested in, or maximum portfolio weight for a fund. We also analyzed portfolios limited to a certain private equity segment (i.e., regional focus, type of investment, or fund management experience). Furthermore, we randomly drew four funds for the relevant vintage year from the TVE dataset for assigning the new commitments to be made in a particular quarter. The new commitments were equally divided among the four funds. To avoid making the results dependent on one particular initial portfolio or on the funds selected for the recommitments, we simulated 1,000 portfolios for each vintage year and averaged the results.

We evaluated the quality of the recommitment strategies by considering various properties of the investment degree—in particular, its mean and standard deviation, as well as the frequency at which overinvesting or liquidity shortfall occurs (i.e., an investment degree that exceeds 1). In computing average portfolio statistics, we discarded the first three years of the portfolio's life to avoid undue influence from the initial portfolio formation period.

Main Results. **Table 3** shows the investment degree characteristics at different ages of the private equity portfolio (ranging from 1 to 26 years), computed for all vintage years. For Strategy I, which sets current commitments equal to current distributions, the average investment degree across all ages is 0.69, which is well below the target level of 1. This finding does not come as a surprise because the committed capital is not called instantaneously, and thus the portfolio always contains a certain amount of cash. The investment degree initially increases after the inception of the portfolio, reaching 0.86 after five years. This increase, however, is followed by a gradual decline to a considerably lower level of just over 0.60 after 20 years. Furthermore, we observe a probability of liquidity shortfall of 4%, on average, although an investment degree in excess of 1 seemingly cannot occur under Strategy I by design. This is due to the overcommitment applied during the formation of the initial portfolio, as becomes clear from the pattern of the probability across fund ages. The probability of overinvestment increases to 22% during the first five years after formation but then declines rapidly again, reaching zero for portfolios older than 10 years.

To better understand the dynamics of the private equity portfolio resulting from this recommitment strategy, Panel A of **Figure 3** shows how the average investment degree evolves over time for our earliest vintage year (1980). For 1980, in fact, the average investment degree comes very close to the target level of 1 in Years 2 and 3 because of the overcommitment in the initial portfolio.

The first recommitment strategy suffers from two problems that result in an average investment degree below the target level of 1. First, the committed capital is not called instantaneously but has a delay that can extend up to several years. Second, part of the committed capital is never called at all. The results from the second recommitment strategy suggest that the first problem is the more important one. Recall that Strategy II attempts to remedy the second problem by increasing the commitments at time t with uncalled capital from the commitments made at $t - 24$. We can see in Table 3 that this scenario results in only a modest increase in the average investment degree, from 0.69 to 0.72. As expected, the improvement starts approximately six years after the initial portfolio formation, as shown in Figure 3 (Panel A).

The results for Strategy III clearly underscore the importance of applying a dynamic overcommitment strategy. Using the investment degree of the existing portfolio for setting the overcommitment percentage for the current recommitments appears to be quite effective because it increases the average investment degree and reduces its variation. Table 3 shows that the average investment degree of Strategy III is 0.86, well above the level attained with Strategies I and II. Not surprisingly,

Table 3. Summary Statistics of the Investment Degree in Recommitment Strategies I–III across Portfolio Ages

Age (year)	Strategy I			Strategy II			Strategy III		
	Mean	StdDev	$P_{(ID>1)}$	Mean	StdDev	$P_{(ID>1)}$	Mean	StdDev	$P_{(ID>1)}$
1	0.36	0.15	0%	0.35	0.14	0%	0.35	0.14	0%
2	0.60	0.18	2	0.60	0.18	2	0.60	0.18	2
3	0.78	0.20	14	0.77	0.20	14	0.79	0.19	14
4	0.84	0.18	19	0.84	0.18	19	0.87	0.17	20
5	0.86	0.17	22	0.86	0.17	22	0.90	0.15	24
6	0.84	0.16	15	0.84	0.16	15	0.89	0.14	18
7	0.80	0.13	7	0.82	0.14	8	0.88	0.11	11
8	0.77	0.12	4	0.80	0.13	5	0.87	0.11	8
9	0.74	0.11	2	0.78	0.12	3	0.87	0.11	7
10	0.73	0.10	1	0.77	0.10	1	0.86	0.10	6
11	0.71	0.09	0	0.75	0.09	0	0.85	0.09	4
12	0.70	0.08	0	0.74	0.08	0	0.84	0.09	3
13	0.66	0.07	0	0.70	0.08	0	0.82	0.10	3
14	0.65	0.06	0	0.69	0.07	0	0.82	0.08	3
15	0.65	0.06	0	0.69	0.06	0	0.82	0.09	3
16	0.64	0.06	0	0.68	0.06	0	0.82	0.08	3
17	0.63	0.06	0	0.67	0.06	0	0.83	0.09	3
18	0.63	0.06	0	0.67	0.06	0	0.83	0.09	3
19	0.63	0.05	0	0.67	0.06	0	0.84	0.08	3
20	0.63	0.06	0	0.67	0.06	0	0.85	0.08	3
21	0.62	0.06	0	0.66	0.06	0	0.85	0.09	5
22	0.61	0.06	0	0.66	0.06	0	0.86	0.09	6
23	0.61	0.05	0	0.66	0.06	0	0.88	0.08	6
24	0.61	0.05	0	0.67	0.06	0	0.90	0.07	8
25	0.61	0.05	0	0.67	0.05	0	0.92	0.06	9
26	0.61	0.05	0	0.67	0.05	0	0.92	0.07	11
Average	0.69	0.09	4	0.72	0.09	4	0.86	0.10	8

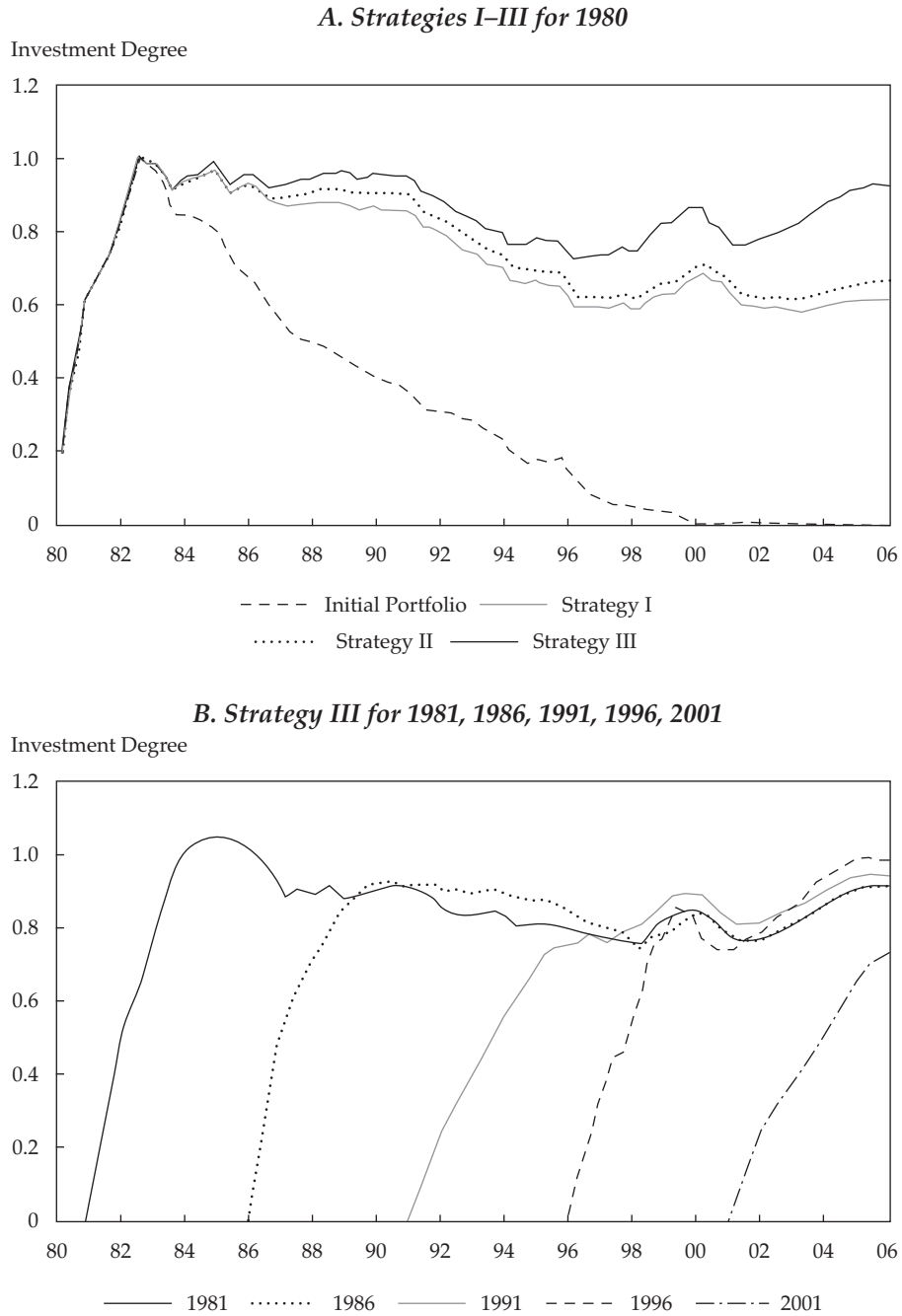
Notes: This table shows properties of the investment degree for private equity portfolios maintained by using recommitment Strategies I, II, and III by age. Strategy I sets current commitments equal to current distributions, Strategy II sets current commitments equal to current distributions plus uncalled commitments, and Strategy III sets current commitments equal to current distributions plus uncalled commitments divided by the investment degree. Reported are the mean, the standard deviation (StdDev), and the fraction of observations with an investment degree higher than 1 [$P_{(ID>1)}$]. Age statistics are based on 1,000 simulated portfolios for each vintage year (1980–2005). In each simulation, the initial portfolio is composed of 16 randomly selected funds from the relevant vintage year. Quarterly recommitments in subsequent years are equally distributed among four randomly selected new funds from that year. The mean excludes the portfolio initialization period (Years 1, 2, and 3).

this result comes at the cost of a higher risk of being overinvested, although the increase in the probability of liquidity shortfall is quite modest (from 4% to 8%). Note also that the range of the average investment degree across the ages of the portfolio is smaller, between 0.82 and 0.92. In addition, both the volatility of the investment degree and the probability of liquidity shortfall are lower for portfolios older than 10 years. These results suggest that the investment degree becomes more stable as the portfolio matures.

A different perspective that confirms the stability of our results is provided by Panel B of Figure 3, which shows time-series behavior of the average investment degree for selected vintage year portfolios (1981, 1986, 1991, 1996, and 2001). We can see that the average investment degrees behave similarly once the portfolios mature, which indicates that the results are genuine outcomes of the strategy and are not due to the particularities of a specific vintage year. For example, for all vintage years, the investment degree declines in 2000, driven by the large distributions made during the dot-com bubble in that year. Nevertheless, the investment degree quickly increases again in subsequent years.

The results from Strategy III suggest that a dynamic overcommitment strategy is effective in bringing the investment degree close to the target level of 1, with only a modest risk of becoming overinvested.

Figure 3. Average Investment Degree of Private Equity Fund Portfolios under Strategies I–III for 1980 and Strategy III for 1981, 1986, 1991, 1996, and 2001

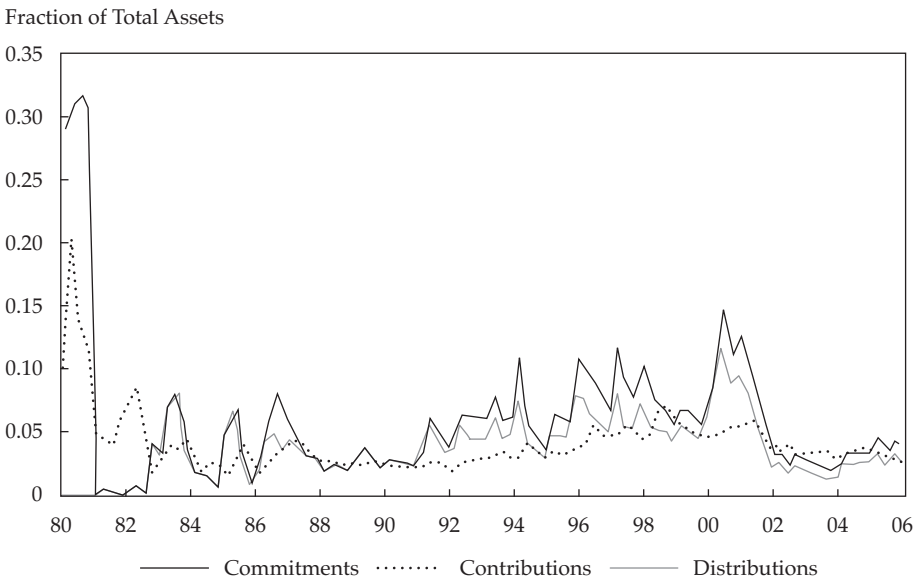


Economic and Institutional Conditions. The performance of recommitment strategies is likely to vary depending on the drawdown and distribution rates of the funds, which, in turn, depend on economic and institutional conditions, at least to some extent. From Gompers and Lerner (1998, 2000), Kaplan and Schoar (2005), and Gompers et al. (2009), we know that both capital flows and returns in the private equity market are cyclical. For example, the venture capital market experienced booms in 1981–1983 and in 1998–2000, when investments grew dramatically in personal computer hardware manufacturers and in internet and telecommunication companies, respectively. This observation gives rise to the question whether our recommitment strategy is cyclical in nature. For

example, we might be committing aggressively when the market becomes overvalued because we will receive more distributions than usual. Moreover, because of dynamic overcommitment, we will likely make larger commitments when investments are difficult to find and cash levels in the portfolio are growing. Both effects can lead to an undesirable cyclical accumulation of new commitments. A detailed picture of the cash flows involved in Strategy III, provided by **Figure 4** for the 1980 portfolios, suggests that the strategy is not highly cyclical. On average, the distributions amount to 5% of the total portfolio value per quarter. Actual investments (contributions) are slightly lower than 5% but are much more constant than the distributions. The new commitments, however, show some cyclical behavior in, for example, 2000. Nevertheless, the stability of the actual contributions illustrates that the cyclicality of our strategy is limited. Although we can see a rise in the commitments in Year 7 because of the recommitments of the uncalled capital of the initial portfolio, this effect does not recur at later stages. This finding also holds for other vintage years.[15] The potential for cyclical behavior in our portfolio is small and not a major issue because our aim is to achieve a passive exposure to the private equity market that includes investments during over- and undervalued periods. A possible explanation for our strategy's lack of cyclicality might be the use of international data in our empirical analysis, leading to diversification over economic cycles (as well as over institutional conditions).

With respect to venture capital, Cumming, Schmidt, and Walz (2010) demonstrated that cross-country differences in institutional conditions have a strong influence on the rate of investment, the ability of a fund to properly manage deal flow, and deal financing. These effects are reported to exist apart from economic conditions. Better legal and accounting standards lower the costs of information flows and, therefore, facilitate faster deal screening and deal origination. Our international dataset, which includes U.S. and European funds, might affect our recommitment strategy because of the reported link between the quality of the legal system and the drawdown rate (i.e., commitments in one country might be called faster than in another country). Although our dataset was less granular than that of Cumming, Schmidt, and Walz, we did observe a difference in the drawdown rate for U.S. and European venture capital funds in our data sample. A closer look at our data reveals that it takes, on average, 16 quarters to call 80% of the commitment for U.S. funds, compared with 22 quarters for European funds. This finding corroborates the results reported by Cumming, Schmidt, and Walz (2010). Because we had cash flow data only at the fund level and not at the deal level, we were unable to make a more detailed analysis of this issue. This observation, however, motivated us to analyze our recommitment strategy on a regional level.

Figure 4. (Re)Commitments and Cash Flows of Strategy III for 1980



Restricted Portfolios for Strategy III. We evaluated Strategy III after imposing restrictions on portfolios with respect to private equity segment (VC or BO) and region (United States or EU). We also evaluated Strategy III for portfolios restricted to first-time funds or follow-on funds. First-time funds are typically not in a position to turn away new investors, whereas established private equity fund managers may restrict access to their follow-on funds. Access to follow-on funds is often limited to institutions that already participate in a current fund. Moreover, first-time funds invest differently than seasoned funds (Ljungqvist, Richardson, and Wolfenzon 2007).

Figure 5 plots the average investment degree for the 1986 portfolios restricted to VC, BO, U.S., EU, first-time, and follow-on funds, together with the unrestricted portfolio. Vintage year 1986 has the longest available history for all the various segments. Panel A of **Table 4** shows the corresponding summary statistics. The average investment degree for BO (0.89) and VC (0.85) portfolios is similar to that of the unrestricted portfolio (0.86). The volatility of the investment degree strategies is slightly higher for VC (0.12) than for BO (0.09). Moreover, the investment degrees of the unrestricted and VC portfolios show similar time-series behavior, as can be seen in Figure 5. This close resemblance can be explained by the distribution of funds over the two investment segments: VC funds constitute two-thirds of the TVE dataset. The difference in investment degree between VC and BO portfolios is particularly clear during the dot-com bubble in 2000–2001. In that period, VC funds made large distributions but the buyout distributions were less extreme.

The results for U.S. portfolios closely resemble those for the unrestricted portfolios, although the average investment degree is slightly lower (0.82) for the former. The average for European portfolios (0.95) is considerably closer to 1. At the same time, the probability of an investment degree above 1 is substantially higher for European portfolios, at 33%, than for U.S. portfolios (only 4%).

The average investment degree for follow-on fund portfolios (0.83) is similar to that of unrestricted portfolios (0.86), whereas the average investment degree for first-time fund portfolios (0.93) is higher. From Figure 5, we can see that the average investment degrees for unrestricted and first-time portfolios deviate the most, with the difference being most observable during the dot-com bubble in 2000–2001. During that period, many first-time VC funds were started. The volatility of the investment degree of the follow-on fund portfolios (0.10) is similar to that of the total sample (0.09), whereas the investment degree of the first-time portfolios is somewhat more volatile (0.12). Furthermore, the small differences between portfolios limited to first-time funds or follow-on funds suggest that the impact of (restrictions on) co-investing, as discussed previously, is limited. First-time funds are not subject to co-investing from earlier funds of the same management firm.

Figure 5. Average Investment Degree of Private Equity Fund Portfolios under Strategy III for 1986 and Restricted Portfolios

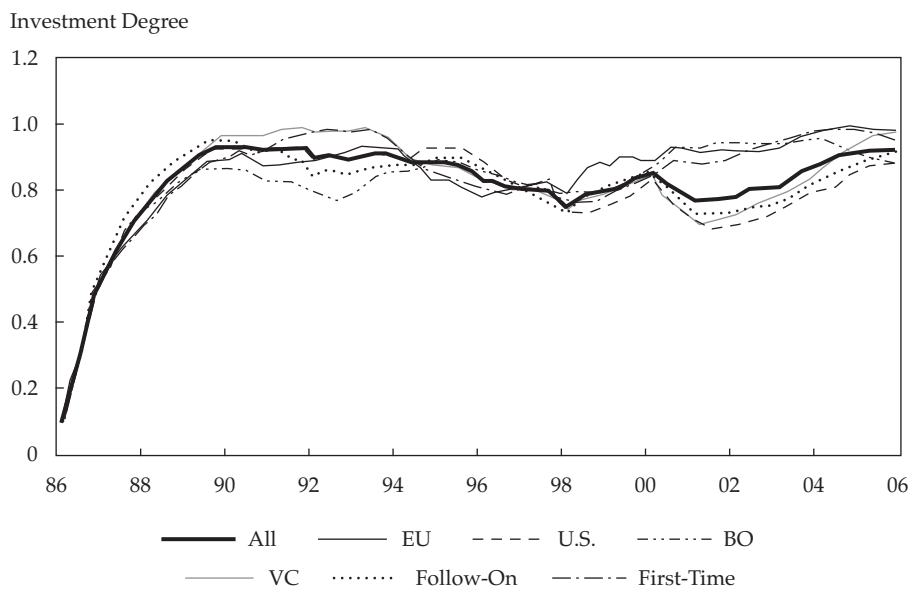


Table 4. Summary Statistics of the Investment Degree in Recommitment Strategies—Robustness Checks

	Mean	StdDev	$P_{(ID>1)}$
A. Restricted portfolios			
Europe	0.95	0.13	33%
United States	0.82	0.09	4
Buyout capital	0.89	0.09	11
Venture capital	0.85	0.12	12
First-time funds	0.93	0.12	26
Follow-on funds	0.83	0.10	5
B. Additional overcommitment			
10%	0.90	0.11	18%
20%	0.94	0.12	31
30%	0.98	0.13	43
40%	1.03	0.14	53
50%	1.07	0.16	66

Notes: This table shows properties of the investment degree for private equity portfolios whose current commitments are set equal to current distributions plus uncalled commitments divided by the investment degree; specific portfolios consist of only European, U.S., venture capital, buyout capital, follow-on, or first-time funds. Reported is the mean investment degree based on 1,000 simulated portfolios (excluding the first three years of the portfolios' lives). In each simulation, the initial portfolio comprises 16 randomly selected funds from the relevant vintage year. Quarterly recommitments in subsequent years are equally distributed among four randomly selected new funds from that year. The first four years for Europe and five years for buyout capital are missing because not enough funds were available.

These results indicate that the use of a data sample restricted to a specific segment, region, or manager experience, as opposed to an unrestricted sample, does not materially affect the quality of Strategy III. This finding convincingly establishes the robustness of our results.

Additional Overcommitment in the Recommitment Phase. Thus far, our analysis has demonstrated that using dynamic overcommitment based on the investment degree of the current private equity portfolio leads to a stable allocation that is fairly close to the target. Nevertheless, the resulting average private equity exposure of 0.86 may still be considered too far below the target. Additional overcommitment can increase the average investment degree, but this increase presumably comes at a greater risk of being overinvested. Therefore, we reconsidered Strategy III after increasing the overcommitment by a constant percentage (OC) equal to 10%, 20%, . . ., 50%—that is, new commitments are given by

$$C_t = \frac{1+OC}{ID_t}\left(D_t + UC_{t-P}\right). \tag{10}$$

Figure 6 plots the average investment degrees resulting from these strategies for the 1980 portfolios, and summary statistics across all vintage years are provided in Panel B of Table 4. Inflating the overcommitment percentage appears to be effective, in the sense that the average investment degree goes up when the additional overcommitment increases. The increase in the investment degree for the 1980 portfolios in Figure 6 is also prevalent for the other vintage years.[16] The average investment degree increases from 0.86 to 0.90, 0.94, 0.98, 1.03, and 1.07 with an additional overcommitment equal to 10%, . . ., 50%, respectively. Not surprisingly, the accompanying probability of being overinvested increases substantially—to 18%, 31%, 43%, 53%, and 66%—whereas it is only 9% for Strategy III without an additional overcommitment. Hence, a more aggressive overcommitment strategy is seemingly suitable only when liquidity shortfall is not a serious problem for an institutional investor, which may be the case when private equity is part of a larger investment portfolio that also includes liquid assets that can be sold (temporarily) to provide the cash necessary for the private equity investments. Siegel (2008), however, clearly showed that investors should be cautious with "too large" allocations to private equity and other illiquid asset classes. Portfolio liquidity should be stress-tested against catastrophic market scenarios because private equity allocations tend to rise in bear markets and could completely dominate the portfolio.

Allocation Changes. Strategic allocations to private equity vary considerably from investor to investor because of differences in investment objectives and liquidity requirements. Foundations and endowments are reported to have larger allocations to private equity (12%) than pension funds (6%) and insurance companies (3%).[17] In our study, we assumed that the decision to pursue a certain private equity exposure had already been made. Owing to the lack of consensus regarding return (Phalippou and Gottschalg 2009) and risk (Cumming, Hass, and Schweizer 2010), however, relatively little guidance on private equity allocations can be found in the literature. Moreover, Siegel (2008) strongly suggested considering liquidity constraints in constructing the strategic portfolio because many of today's most popular asset classes appear to be illiquid. As a result, the target allocation to private equity might change over time. This possibility motivated us to test how changes in the target allocation affect our recommitment strategy.

To study the impact of an allocation change, we considered the situation whereby the 1980 portfolio receives a 20% inflow in 1986. **Figure 7** plots

Figure 6. Average Investment Degree of Private Equity Fund Portfolios under Strategy III with Varying Degrees of Additional Overcommitment for 1980

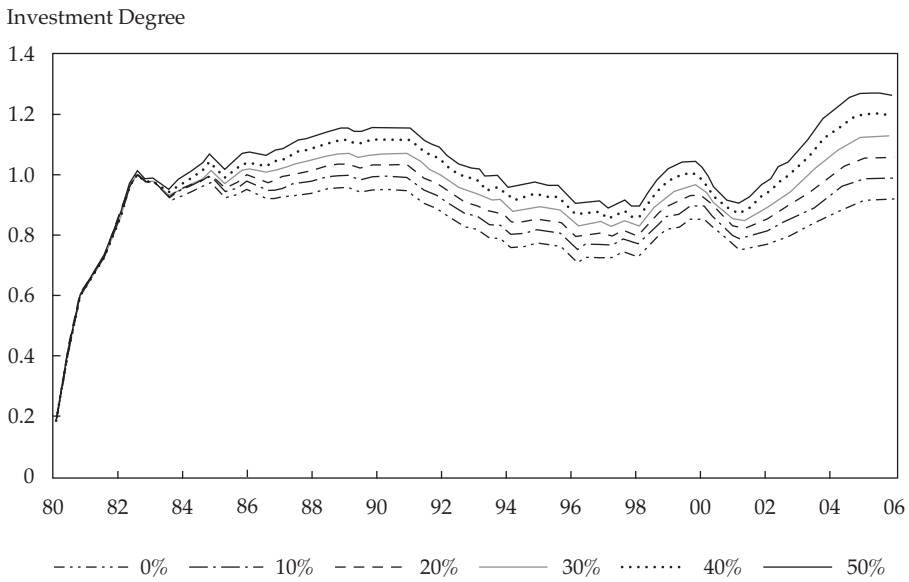
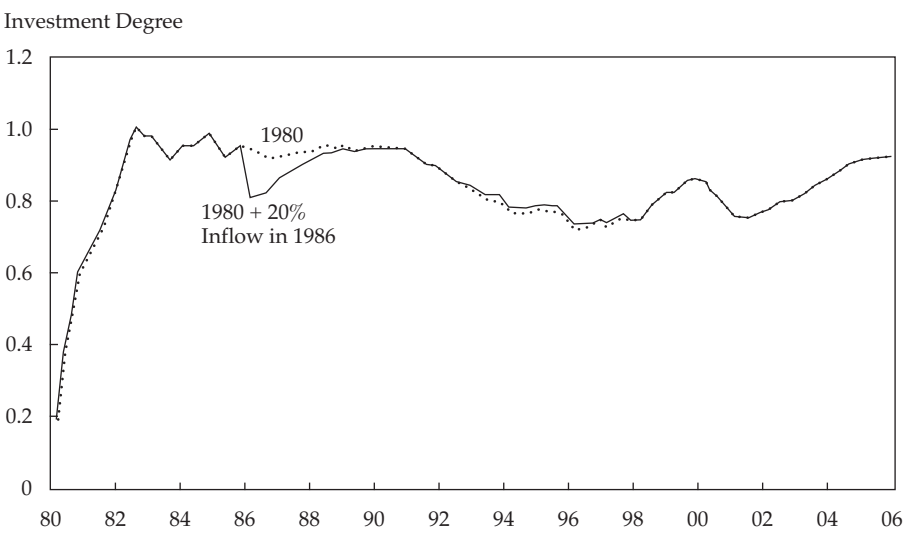


Figure 7. Average Investment Degree of Private Equity Fund Portfolios under Strategy III for 1980 with and without a 20% Inflow in 1986



the time series of the average investment degree for this portfolio, together with the same 1980 portfolio but without additional inflow. The graph clearly shows a drop in the investment degree in 1986. More importantly, we can see that the investment degree already converges back to the initial level around three years after the change in allocation. Unquestionably, a 20% inflow is an arbitrary choice, but this example is representative of the effect of an inflow. Initially, the investment degree will decrease, followed by a recovery that will take about three years. Overall, this finding suggests that our strategy is very well able to deal with changes in the strategic asset allocation.

Conclusion

In our study, we developed a recommitment strategy for private equity funds that is capable of maintaining a stable private equity portfolio. The key feature of our dynamic strategy is that the level of new commitments in a given period depends on the characteristics of the current private equity

portfolio. New commitments are set equal to last quarter's received distributions plus the uncalled capital from old commitments, scaled by the ratio of the target allocation to its current private equity allocation. In a historical simulation exercise, this strategy, on average, realized 86% of the target allocation while keeping the probability of being overinvested low (8%). Our sensitivity analyses showed that our strategy remains successful when the portfolio is restricted to a certain private equity segment (buyout or venture capital), specific region (the United States or Europe), or varying fund manager experience (first-time or follow-on funds). Furthermore, we showed that an investor who can permit a temporary higher allocation should consider a structural 30% additional overcommitment, which will bring the portfolio exposure closer to the target (98%) but at the cost of a higher risk of being overinvested (43%).

Our recommitment strategy may be useful to institutional investors that have included private equity in their strategic policy portfolio, especially those with high allocations to illiquid assets. The dynamic overcommitment strategy will commit more aggressively when the current portfolio is underinvested but will reduce new commitments when the portfolio is overinvested. In the case of overinvesting, investors might not be able to meet a capital call owing to a cash shortage. At that point, the investors face involuntary liquidation of their fund investments such that they are forced to sell their fund position at a large discount in the secondary market. This illiquidity problem actually became real during the recent liquidity crisis (see Siegel 2008). Our dynamic strategy limits this undesirable situation as much as possible.

Because recommitment strategies have received very little attention in the literature, we encourage further research in this area. A prominent issue that requires attention concerns private equity recommitments in the context of a larger investment portfolio that includes other asset classes. The dynamics of such a portfolio will differ from those of the 100% private equity portfolio that we considered in our study because stocks and bonds do not perfectly co-vary with the private equity investments. In addition, more accurate intermediate valuations of existing investments—using, for example, the methodology presented in Driessen, Lin, and Phalippou (2007)—might further improve the recommitment strategy. Most fund managers value their investments at cost during the first years of their investments. This valuation methodology could bias valuations downward. In the next period, fund managers have a stronger incentive to strategically overvalue their portfolio companies in order to attract new institutional investors for their successor funds (Cumming and Walz 2010). Both over- and undervaluation affect our recommitment strategy through the scaling factor, which is equal to the total portfolio value relative to the value of the current private equity portfolio, such that overvaluation could lead to recommitments that are too low and undervaluation could lead to recommitments that are too high. Finally, comparing our single-period recommitment solution to the solution of a multiperiod optimization problem would be interesting.

We are grateful to Tjeert Keijzer, Angelien Kemna, Erik Kole, Josh Lerner, Ludovic Phalippou, Per Strömberg, Joacim Tag, and Marno Verbeek for their helpful suggestions. We also benefited from feedback received at the Inquire Europe meeting held in Stockholm, October 2007; the EFM Symposium on Entrepreneurial Finance and Venture Capital held in Montreal, April 2010; and seminars at Robeco and ABP. We thank Jackie Koshinski for her editing advice. An earlier version of this article was called "A Recommitment Strategy for Long-Term Private Equity Fund Investors."

This article qualifies for 1 CE credit.

Appendix A. Data Methodology

The dataset obtained from Thomson Venture Economics contains information on 2,786 private equity funds over 1980:Q1–2005:Q4 and includes the regional focus (United States or Europe), type of investment (venture capital, buyout capital, mezzanine finance, or fund of funds), vintage year, quarterly contributions and distributions, and quarterly information on the NAV. Reported cash flows are given in U.S. dollars and are net of (management) fees as well as carried interest. In total, we excluded 168 funds on the following grounds:

1. *Total commitments.* The fund's cash flows and NAVs are expressed relative to its total commitment, which makes funds of different sizes comparable. One fund reported a zero commitment and was excluded from the dataset.
2. *Geographic orientation.* One fund was included in both the European and the U.S. samples. The double counting was excluded and the fund was characterized as European.
3. *Type of investment.* Mezzanine funds (65 funds) were removed because their structures differ from those of private equity funds. Because this research focuses on private equity fund investors, data on funds of funds (direct investing [13 funds] and secondaries [7 funds]) were also excluded.

4. *Missing observations.* Two funds reported cash flows equal to zero over the entire period and were, therefore, excluded.
5. *Visual inspection.* Seventy-one funds were removed upon visual inspection of the data.

The Thomson Venture Economics database reports a fund's contributions, distributions, and estimated NAVs. The contributions and distributions, if any, are assumed to occur at the end of the month, and information on the NAVs is given on a quarterly basis. The following adjustments were made to these cash flow variables:

1. One hundred fifty-seven funds reported negative contributions, which were changed to distributions.
2. Negative distributions of 14 funds were adjusted by subtracting them from the funds' earlier distributions.
3. Eight funds reported a negative NAV. Because the NAVs of funds are highly unlikely to become negative, these funds were removed.

Notes

1. Increasingly, the largest institutional investors (e.g., the Ontario Teachers' Pension Plan, the Government of Singapore Investment Corporation, and Temasek) believe it is worth the time and effort to make a direct private equity effort and eliminate the middleman or at least lower the fees they must pay.
2. Several studies have examined which characteristics make private equity managers successful, including expertise (Cumming, Fleming, and Suchard 2005), experience (Sørensen 2007), specialization (Gompers, Kovner, Lerner, and Scharfstein 2009), and network (Hochberg, Ljungqvist, and Lu 2007).
3. Lerner and Schoar (2004) showed that restrictions on the transfer of fund ownership are used by young funds and funds that focus on industries with longer investment cycles to attract deep-pocket investors (i.e., investors with a low probability of facing a liquidity shock). This approach makes fund-raising for a follow-on fund easier because these investors have an increased probability of continued participation, which can attract new investors. Although this setup of the private equity market structure looks more complex than public equity, Axelson, Strömberg, and Weisbach (2009) demonstrated that the financial structure of private equity funds is optimal for three industry characteristics: (1) pooling of investments, (2) nonlinear profit sharing with the fund manager to limit governance problems, and (3) a financial structure that combines *ex post* fund-raising and specific deal financing. Although the secondary market has grown steadily over time, with current turnover estimated at $15 billion–$20 billion (Lutyens 2008), it is still relatively immature and does not always provide liquidity when investors need it (Franzoni, Nowak, and Phalippou, forthcoming).
4. See de Zwart, Frieser, and van Dijk (2012) for a comparison of static and dynamic private equity recommitment strategies.
5. Kaplan and Schoar (2005) pointed out that voluntary reporting may lead to a selection bias for better-performing funds. Indeed, Phalippou and Gottschalg (2009) found evidence that worse-performing funds are underrepresented. This selection bias is mainly relevant for studies on the (relative) performance of private equity funds. Because the reinvestment strategies in our study were applied to unrestricted portfolios—thus, not relying on the performance of the funds—this bias likely did not influence our conclusions. In addition, the TVE dataset might suffer from a valuation bias and stale pricing. The funds' NAVs, defined as the sum of the NAVs of the individual investee companies, are taken as reported by the fund managers. These NAVs are based on the fund managers' subjective valuation because private equity investments are not evaluated by the market. Furthermore, Cumming and Walz (2010) showed that there are conflicts of interest between managers and investors, which make objective judgment even less likely. Most managers keep the NAV at investment cost during the first years of their investment. This valuation methodology could bias NAVs downward. In the next period, fund managers could have a stronger incentive to overvalue their portfolio companies in order to attract new institutional investors for their successor funds. Cumming and Walz also showed that this strategic overreporting behavior does occur and that less stringent accounting standards and a weak legal framework facilitate overvaluations. Although recent initiatives, such as fair-value guidelines and valuation clauses, may reduce the valuation leeway for fund managers, our historical dataset may suffer from this valuation bias. A simple example shows that substantial differences in valuation have only a small impact on the key measure of our study (i.e., the ratio of the NAV to total assets, or NAV plus cash, which we call the *investment degree*). Suppose that we have an NAV of 90 and a cash amount of 10. Our investment degree is thus 90% [= 90/(90 + 10)]. If the NAV is 20% lower (72), the resulting investment degree would be 88% [= 72/(72 + 10)]. This example illustrates that small differences in the valuation would be unlikely to affect our results significantly.
6. Co-investments are also known as *cross-overs* in the private equity industry.
7. Note that the contributions in our dataset are net of fees.
8. The size of a private equity fund is defined as the sum of all the investors' commitments to the fund.
9. Phalippou and Gottschalg (2009) showed that writing off these "living-dead" investments lowers the average private equity returns. Because we focused on cash flows and not on returns, this adjustment had little impact on our analysis. The results (including the residual NAV values), which are available upon request, are qualitatively similar.
10. The variation in the timing of the maximum investment degree across vintage years and the variation in the distributions after four years reflect the fluctuations in supply and demand for private equity documented by Gompers and Lerner (1998) and Gompers, Kovner, Lerner, and Scharfstein (2008). Demand for private equity capital appears to be related to changes in investment opportunities that arise from, for example, technological innovations or changing monetary regimes. Furthermore, the supply of private equity capital has also been reported to vary over time owing to changes in regulatory factors (Poterba 1989; Gompers and Lerner 1998; Cumming and Johan 2007) or

labor market rigidities (Jeng and Wells 2000). Also, the cycles in the IPO market contribute to the cyclical behavior of the distributions.

11. Active strategies to select the best funds (see Lerner, Schoar, and Wongsunwai 2007) were not within the scope of our study.
12. Some investors may have an asymmetric utility function. For example, being overinvested is more costly than being underinvested when other assets cannot be sold and credit lines, at additional cost, are needed to fund liquidity shortfall or when overinvesting breaches investment guidelines in a more diversified portfolio setting.
13. The corresponding multiperiod optimization problem involves the loss function

$$\min_{C_t, C_{t+1}, C_{t+2}, \ldots} \mathrm{E}_t \left[\sum_{i=1}^{\infty} \beta^{i-1} \left(1 - ID_{t+i}\right)^2 \right], \tag{11}$$

where $0 \le \beta \le 1$ is a discount factor. If we define Equation 11 as $L(ID_t)$, this can be rewritten as

$$\begin{aligned} L\left(ID_t\right) &= \min_{C_t} \mathrm{E}_t \left\{ \left(1 - ID_{t+1}\right)^2 + \beta \min_{C_{t+1}, C_{t+2}, \ldots} \right. \\ &\quad \left. \mathrm{E}_{t+1} \left[\sum_{i=1}^{\infty} \beta^{i-1} \left(1 - ID_{t+i+1}\right)^2 \right] \right\} \\ &= \min_{C_t} \mathrm{E}_t \left[\left(1 - ID_{t+1}\right)^2 + \beta L\left(ID_{t+1}\right) \right]. \end{aligned} \tag{12}$$

Despite the fact that this is a dynamic programming problem with a quadratic loss function, it is not straightforward to solve owing to the nature of the restrictions, as given by Equations 3, 4, and 5. In particular, the fact that the current commitments C_t affect the capital calls during the next T periods complicates the analysis (see Equation 5).

14. Normally, investors spread their initial commitments over two to three years to benefit from vintage year diversification, whereas a limited number of investors try to buy an existing portfolio in the secondary market or buy listed private equity funds (see Cumming, Fleming, and Johan 2011). The secondary market is not an open market and is not very deep because many funds restrict the transfer of fund ownership (Lerner and Schoar 2004). To examine the relevance of this issue, we also conducted an empirical analysis (discussed later in the article) with initial portfolios built up over two or three years; the investment degrees differ during the first few years, when the portfolios are invested more slowly. After about five years, all the portfolios converge, suggesting that the construction of the initial portfolios does not seem to affect the quality of the recommitment strategies after the portfolios mature. Detailed results are available upon request.
15. Detailed results for other vintage years are available upon request.
16. Detailed results are available upon request.
17. These estimates are as of December 2010 and were provided by Preqin (2011).

References

Axelson, Ulf, Per Strömberg, and Michael S. Weisbach. 2009. "Why Are Buyouts Levered? The Financial Structure of Private Equity Funds." *Journal of Finance*, vol. 64, no. 4 (August): 1549–1582.

Cardie, J. Heath, Katherine A. Cattanach, and Mary F. Kelley. 2000. "How Large Should Your Commitment to Private Equity Really Be?" *Journal of Wealth Management*, vol. 3, no. 2 (Fall):39–45.

Cumming, Douglas J., and Sofia A. Johan. 2007. "Regulatory Harmonization and the Development of Private Equity Markets." *Journal of Banking & Finance*, vol. 31, no. 10 (October): 3218–3250.

Cumming, Douglas J., and Uwe Walz. 2010. "Private Equity Returns and Disclosure around the World." *Journal of International Business Studies*, vol. 41, no. 4 (May):727–754.

Cumming, Douglas J., Grant Fleming, and Sofia A. Johan. 2011. "Institutional Investment in Listed Private Equity." *European Financial Management*, vol. 17, no. 3 (June):594–618.

Cumming, Douglas J., Grant Fleming, and Jo-Ann Suchard. 2005. "Venture Capitalist Value-Added Activities, Fundraising and Drawdowns." *Journal of Banking & Finance*, vol. 29, no. 2 (February):295–331.

Cumming, Douglas J., Lars H. Hass, and Denis Schweizer. 2010. "Private Equity Benchmarks and Portfolio Optimization." Working paper (October).

Cumming, Douglas J., Daniel Schmidt, and Uwe Walz. 2010. "Legality and Venture Capital Governance around the World." *Journal of Business Venturing*, vol. 25, no. 1 (January):54–72.

de Malherbe, Etienne. 2004. "Modeling Private Equity Funds and Private Equity Collateralised Fund Obligations." *International Journal of Theoretical and Applied Finance*, vol. 7, no. 3 (May):193–230.

de Zwart, Gerben, Brian Frieser, and Dick van Dijk. 2012. "When Rules of Thumb Fail: Static Versus Dynamic Recommitment Strategies for Institutional Private Equity Investors." Working paper, Erasmus University Rotterdam.

Driessen, Joost, Tse-Chun Lin, and Ludovic Phalippou. 2007. "Estimating the Performance and Risk Exposure of Private Equity Funds: A New Methodology." Netspar Discussion Paper 2007 – 023 (August).

Franzoni, Francesco, Eric Nowak, and Ludovic Phalippou. Forthcoming. "Private Equity Performance and Liquidity Risk." *Journal of Finance*.

Gompers, Paul A., and Josh Lerner. 1996. "The Use of Covenants: An Analysis of Venture Partnership Agreements." *Journal of Law & Economics*, vol. 39, no. 2 (October):463–498.

———. 1998. "What Drives Venture Capital Fundraising?" *Brookings Papers on Economic Activity: Microeconomics*:149–192.

———. 1999. *The Venture Capital Cycle*. Cambridge, MA: MIT Press.

———. 2000. "Money Chasing Deals? The Impact of Fund Inflows on Private Equity Valuations." *Journal of Financial Economics*, vol. 55, no. 2 (February):281–325.

Gompers, Paul A., Anna Kovner, Josh Lerner, and David Scharfstein. 2008. "Venture Capital Investment Cycles: The Impact of Public Markets." *Journal of Financial Economics*, vol. 87, no. 1 (January):1–23.

———. 2009. "Specialization and Success: Evidence from Venture Capital." *Journal of Economics & Management Strategy*, vol. 18, no. 3 (Fall):817–844.

Hochberg, Yael V., Alexander Ljungqvist, and Yang Lu. 2007. "Whom You Know Matters: Venture Capital Networks and Investment Performance." *Journal of Finance*, vol. 62, no. 1 (January):251–301.

Jeng, Leslie A., and Philippe C. Wells. 2000. "The Determinants of Venture Capital Funding: Evidence Across Countries." *Journal of Corporate Finance*, vol. 6, no. 3 (September):241–289.

Jones, Charles M., and Matthew Rhodes-Kropf. 2003. "The Price of Diversifiable Risk in Venture Capital and Private Equity." Working paper, Columbia University (May).

Kaplan, Steve, and Antoinette Schoar. 2005. "Private Equity Performance: Returns, Persistence, and Capital Flows." *Journal of Finance*, vol. 60, no. 4 (August):1791–1823.

Lerner, Josh, and Antoinette Schoar. 2004. "The Illiquidity Puzzle: Theory and Evidence from Private Equity." *Journal of Financial Economics*, vol. 72, no. 1 (April):3–40.

Lerner, Josh, Antoinette Schoar, and Wan Wongsunwai. 2007. "Smart Institutions, Foolish Choices? The Limited Partner Performance Puzzle." *Journal of Finance*, vol. 62, no. 2 (April):731–764.

Ljungqvist, Alexander, and Matthew Richardson. 2003. "The Investment Behaviour of Private Equity Fund Managers." Working paper, London School of Economics (October).

Ljungqvist, Alexander, Matthew Richardson, and Daniel Wolfenzon. 2007. "The Investment Behavior of Buyout Funds: Theory and Evidence." Working paper, New York University (June).

Lutyens, Campbell. 2008. "The Private Equity Secondary Market." PEI Media Ltd.

Nevins, Daniel, Andrew Conner, and Greg McIntire. 2004. "A Portfolio Management Approach to Determining Private Equity Commitments." *Journal of Alternative Investments*, vol. 6, no. 4 (Spring):32–46.

Phalippou, Ludovic, and Oliver Gottschalg. 2009. "The Performance of Private Equity Funds." *Review of Financial Studies*, vol. 22, no. 4 (April):1747–1776.

Poterba, James M. 1989. "Venture Capital and Capital Gains Taxation." In *Tax Policy and the Economy*, vol. 3 (May). Edited by Lawrence H. Summers. Cambridge, MA: MIT Press.

Preqin. 2011. "2011 Preqin Global Private Equity Report."

Sahlman, William A. 1990. "The Structure and Governance of Venture-Capital Organizations." *Journal of Financial Economics*, vol. 27, no. 2 (October):473–521.

Sharpe, William F. 2010. "Adaptive Asset Allocation Policies." *Financial Analysts Journal*, vol. 66, no. 3 (May/June):45–59.

Siegel, Laurence B. 2008. "Alternatives and Liquidity: Will Spending and Capital Calls Eat Your Modern Portfolio?" *Journal of Portfolio Management*, vol. 35, no. 1 (Fall):103–114.

Sørensen, Morton. 2007. "How Smart Is Smart Money? A Two-Sided Matching Model of Venture Capital." *Journal of Finance*, vol. 62, no. 6 (December):2725–2762.

Takahashi, Dean, and Seth Alexander. 2002. "Illiquid Alternative Asset Fund Modeling." *Journal of Portfolio Management*, vol. 28, no. 2 (Winter):90–100.

Weidig, Tom, Andreas Kemmerer, and Björn Born. 2005. "The Risk Profile of Private Equity Funds of Funds." *Journal of Alternative Investments*, vol. 7, no. 4 (Spring):33–41.